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INVENTORIES, RATIONAL EXPECTATIONS AND THE BUSINESS CYCLE

Alan A. Blinder and Stanley Fischer

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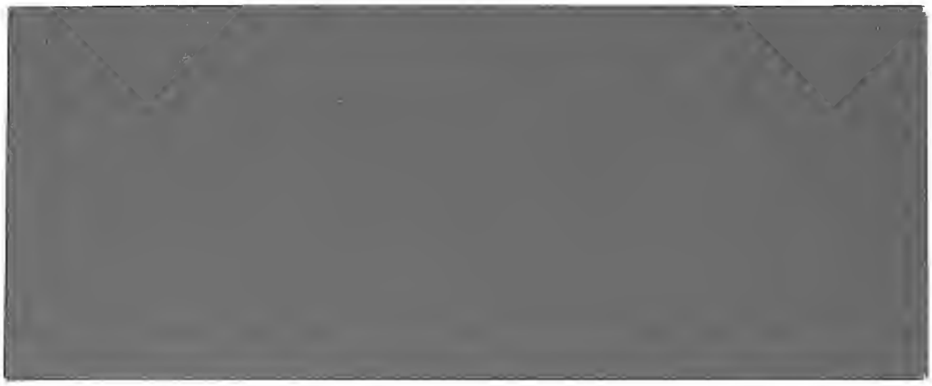

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1. Introduction.

There are doubtless many mechanisms that co-operate in producing the serial correlation of deviations of output from potential output that is known as the business cycle. But even a cursory look at recent data indicates the importance of inventory fluctuations in the short-run dynamics of output. Table 1 shows quarterly changes in both real GNP and its most volatile component--inventory investment--during the 1973-6 recession and recovery. The significance of inventory change is evident. To cite only the most dramatic figures, almost the entire decline in GNP during the worst quarter of the downturn (1975:1) came from a swing in inventory investment from +$7 billion to -$20 billion at annual rates; and about two-thirds of the GNP change in the strongest quarter of the recovery (1975:3) resulted from the end of inventory decumulation. This is not, of course, meant to imply that autonomous movements in inventories cause the business cycle, but only to suggest that inventory dynamics play a fundamental role in its propagation. This paper studies the role of inventories in the business cycle in a model with rational expectations.[1]

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1. For a more complete, but similar, analysis of inventory behavior in a conventional nonstochastic macro model without rational expectations, see Blinder (1977).

Table 1

Changes in Real GNP and Real Inventory

Investment, 1973-1976*

	1973:4	1974:1	1974:2	1974:3	1974:4	1975:1	1975:2	1975:3	1975:4	1976:1
Real GNP	+6.3	-12.4	-5.7	-7.6	-17.2	-29.9	+18.4	+32.5	+9.1	+26.2
Real inventory investment	+11.3	-11.5	-4.7	-7.2	+4.8	-26.8	+2.0	+20.9	-7.5	+14.3

* Change from previous quarter at annual rate, in billions of 1972 dollars.

Source: U.S. Department of Commerce

Recent work on business cycles and monetary policy has been greatly influenced by the approach to aggregate supply due to Lucas (1972, 1973). Basing his argument on intertemporal substitution effects, and on the stylized facts that long-run labor supply functions are vertical while short-run functions have positive slope, Lucas has posited the aggregate supply function:

$$(1.1) \qquad y_t = k_t + \gamma(p_t - {}_{t-1}p_t) + e_t,$$

where y is (the log of) real output, k is (the log of) the natural rate of output, p is (the log of) the price level, and the notation ${}_{t-1}X_t$ denotes the expectation that is formed at time (t-1) of the variable X_t. In Lucas' work, and in this paper, these expectations will be assumed to be formed rationally. Finally, the error term, e_t, is assumed to be independently and identically distributed.

If the natural rate of output, the k_t term in (1.1), is exogenous, two strong conclusions follow from coupling this supply function with the assumptions that prices always move to clear markets within the period and that expectations are rational. The first is that deviations of output from its natural rate are pure white noise--there is no business cycle. The second is that no feedback rule for monetary policy (or equivalently, no anticipated change in the money stock) can affect deviations of output from the natural rate.

That both these conclusions follow from rational expectations, price flexibility, and (1.1), can be shown simply. The basic implication of rational expectations is that errors in predicting the (logarithm of the) price level must be uncorrelated with any variable that is known as of time t-1, including, in particular, the previous prediction error.

Letting u_t (unanticipated inflation) denote these prediction errors, equation (1.1) can be written

$$y_t - k_t = \gamma u_t + e_t$$

which is just white noise. By like reasoning, no known monetary rule can cause unanticipated inflation; only monetary surprises can do that. So, if markets clear, equation (1.1) allows for no real effects of anticipated money, given the assumed fixed natural rate of output.

Explanations for the business cycle that build on the supply function (1.1) focus on the determinants of the k_t term. Lucas (1975) has shown that the inclusion of capital in the model will produce serial correlation of output, as unanticipated inflation affects current output and thereby future capital stocks. A similar mechanism has been explored in Fischer (1977b). Sargent (1977) has studied a model in which serial correlation of the natural rate of output follows from labor supply behavior.

Modifications of the basic model to allow for serial correlation of output do not necessarily modify the second conclusion--that anticipated policy actions have no real effects. A role for monetary policy in affecting cyclical behavior may be found by dropping the market clearing assumption, which changes the form of the aggregate supply function.[1] Alternatively, if capital is explicitly included in the model, and it is assumed that the rate of accumulation of capital is directly or indirectly a function of the anticipated rate of inflation, then the behavior of the k_t term in (1.1) can be affected by anticipated monetary changes.[2]

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1. Fischer (1977a) and Phelps and Taylor (1977); however, see also McCallum (1977) for a demonstration that some types of nonmarket clearing still do not permit any role for monetary policy in affecting output.

2. Fischer (1977b).

In this paper we show how the inclusion of storable output affects the two basic conclusions arising from the combination of the supply function (1.1) and rational expectations. First, we show that adding inventories to the model makes shocks persist. Second, we show that if the demand for inventories is interest elastic, anticipated monetary changes have real effects. Of the two roles of inventories, we have no doubt that the propagation of disturbances caused by unanticipated events is much the more important. Nonetheless, it is interesting to note that the inclusion of inventories opens a potential channel for even fully anticipated monetary policy to have real effects.

In the next two sections of the paper we show that, in the presence of storable output, the aggregate supply function is modified to a form like:

$$(1.2) \qquad y_t = k_t + \gamma(p_t - {}_{t-1}p_t) + \lambda(N_t^* - N_t) + e_t$$

where N_t is the stock of inventories at the beginning of the period, and N_t^* is the optimal or desired stock. Section 2 derives a supply function like (1.2) based on utility maximization by a yeoman farmer working in a competitive market, the case that seems closest in spirit to Lucas' analysis.[1] Section 3 derives a similar function in a different setting: that of a profit-maximizing firm with some degree of monopoly power.

The following two sections offer proofs of the assertions we have just made. In Section 4 we show that, even in the most stripped-down macro model with inventories that we can set up, shocks lead to persistent deviations of y_t from its natural level, i.e., to business cycles. And

1. Lucas (1977, p. 18) discusses the way in which the aggregate supply function (1.1) should be modified to take account of inventory behavior, without, however, embodying the modified function in a full model.

Section 5 demonstrates that, if N_t^* depends on the real interest rate, then anticipated changes in the money stock can have real effects through inventory changes. Section 6 contains conclusions.

2. The Lucas Supply Function Revisited: The Case of the Yeoman Farmer.

The Lucas supply function (1.1) is most conveniently thought of as arising from the behavior of individuals selling labor services in isolated markets (Phelpsian islands).[1] Each individual's supply of labor services is an increasing function of the real price (wage) he perceives. However, by virtue of an assumed one-period information lag, he does not know the current aggregate price level, having full information at the time he decides how much labor to supply only on the absolute price (wage) he receives in his isolated market. He faces an inference problem in deciding what real wage is represented by the absolute wage he is offered in his market. Since the price he receives for his service varies from period to period both because the general price level varies and because there are changes in relative prices, he typically ascribes part of any unanticipated increase in the nominal price he faces to a change in the relative price of the service he sells. Accordingly, an unanticipated increase in the absolute price level is misinterpreted as being in part an increase in a relative price, and output is increased. Aggregating over markets, Lucas derives the aggregate supply function (1.1).

Although the Lucas supply function is usually thought of as arising in markets in which services are sold by individuals, much the same derivation applies when nonstorable output is supplied by firms which

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1. See Phelps (1970), Introduction.

hire labor for the purpose of production. Labor may be regarded as distributed randomly to Phelpsian islands, each with many firms, in the current period, with the workers being immobile between islands within the period. Firms' hiring decisions are dependent only on the real wage, expressed in terms of the price of the good they produce. Labor is concerned with the real wage in terms of the average price level.[1] An increase in the price of output in a particular market, observed by both firms and workers, shifts up the labor demand curve (with the nominal wage on the vertical axis) proportionately. However, the workers, concerned with the absolute price level, interpret any increase in price in this market as only in part an increase in the absolute price level. The labor supply curve accordingly moves up less than the demand curve, the nominal wage rises less than proportionately to the increase in the price in this market, and output increases. As before, an unanticipated increase in the general price level will lead to an increase in aggregate output. This is almost precisely the story told by Friedman (1968) in explaining the short-run Phillips curve.

In this section we use a similar framework to examine optimal behavior for a yeoman farmer, working without any cooperating factors, who sells his output in a competitive market. The output is assumed storable and he can therefore obtain goods to sell in two ways: by working, or by drawing down his inventory stocks. At first we assume that the individual knows both the aggregate price level (the average of the prices of things he buys) and the relative price of his own output. Later we allow for confusion between the two.

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1. Both the individual suppliers in the first paragraph and the workers in the second must be assumed to be distributed randomly to other islands to do their shopping after work.

We start with this model not for its realism, but in order to provide micro-foundations for the inclusion of inventories in the macro model. As noted, it is also close in spirit to Lucas' and Phelps' (1970) work. As will become clear, however, inventories work in this model mainly through wealth effects--which is not how we imagine they work in a modern industrial economy. Further, the utility analysis to follow is plagued by the usual ambiguities arising from income and substitution effects. Little beyond the list of arguments for each demand function can in general be derived; meaningful qualitative restrictions on demand and supply functions must generally be assumed, either directly, or else by restricting the class of utility functions. For both these reasons, we deal briefly with this model, and then turn our attention to a model of the firm.

Consider an individual-firm living and working for two periods,[1] producing output according to a linear production function:

$$Y_t = L_t \qquad t = 0,1.$$

He is endowed with beginning-of-period stocks of the good, N_0, and of money, M_0, and must decide how much to produce, how much to consume, and how to carry over his wealth in the two assets available to him: N_1 and M_1. The prices of his own good, and of goods in general, are respectively, W_t and P_t, where we assume that both W_0 and P_0 are known, but W_1 and P_1 are random. We shall work with transformations of W_1 and P_1, namely, next period's relative price, $w_1 = W_1/P_1$, and next period's purchasing power of money, $q_1 = 1/P_1$.

Budget constraints for periods 0 and 1 are:

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1. The two-period set-up is consistent with a multi-period optimization process, in which the second period utility function is the derived utility function for all remaining periods.

$$(2.1) \qquad C_0 = w_0(L_0 + N_0 - N_1) + q_0(M_0 - M_1)$$

$$(2.2) \qquad C_1 = w_1(L_1 + N_1) + q_1M_1.$$

In brief, in period 0 the yeoman farmer decides how much to produce, how much to carry over to period 1 in inventories, and how much to carry over in money. He does this without knowing w_1 or q_1. Then, in period 1, w_1 and q_1 are announced, he decides how much to produce, and then consumes what this output plus his accumulated wealth allow him to.

The individual is assumed to maximize a separable utility function:

$$J = U(C_0, \bar{L} - L_0) + EV(C_1, \bar{L} - L_1),$$

where both U(.) and V(.) are strictly concave, and where any time discounting is embodied in the functional form of V(.).[1] We further assume that consumption and leisure in both periods are normal goods, and that Inada-type conditions on the utility functions preclude solutions with zero consumption or zero leisure.

The period 1 problem is quite simple. With the carry-over stocks predetermined and the two prices known, the problem is one of certainty, with only labor supply to be chosen. That is, the yeoman farmer maximizes:

$$V(w_1L_1 + (w_1N_1 + q_1M_1), \bar{L} - L_1).$$

The first-order condition is the usual one:

$$(2.3) \qquad w_1V_1 - V_2 = 0,$$

where V_i denotes the derivative of V with respect to its i-th argument. Obviously, (2.3) implies a labor supply function with the real wage and real wealth as arguments:

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1. Note again that this set-up is consistent with multi-period optimization.

(2.4) $\quad L_1 = F(w_1, w_1N_1 + q_1M_1).$

F_1 will be positive if the substitution effect dominates the income effect, and F_2 will be negative if leisure is a normal good.

Now, using the budget constraints (2.1) to substitute out for C_0 and C_1, the maximand for the period 0 problem can be written as

(2.5)
$$\max_{\{L_0, q_0M_1, N_1\}} U(w_0(L_0 + N_0 - N_1) + q_0(M_0 - M_1), \bar{L} - L_0)$$
$$+ EV(w_1F(.) + w_1N_1 + q_1M_1, \bar{L} - F(.))$$

subject to $N_1 \geq 0$, $M_1 \geq 0$. Here F(.) is defined in (2.4), and it is convenient to treat real balances (q_0M_1) rather than nominal balances (M_1) as the choice variable. Notice that the second argument of F(.) is

$$w_1N_1 + q_1M_1 = \frac{w_1}{w_0} w_0N_1 + \frac{q_1}{q_0} q_0M_1$$

Next, by examing (2.4) and (2.5), and by observing that $w_1F(.) = w_0(w_1/w_0)F(.)$, it is clear that the arguments of the demand functions for L_0, q_0M_1, N_1 (and therefore also for C_0) must be:

(i) w_0, the current wage or relative price;

(ii) $w_0N_0 + q_0M_0$, real wealth;

(iii) the distributions of the returns on money, q_1/q_0, and on inventory holdings, w_1/w_0.

The absolute price level is not an independent argument in the behavioral functions, entering only to deflate the nominal value of money balances.

For subsequent use we are interested particularly in the supply function for labor (which is also the supply function for output) and the demand function for inventories. We write these as:

(2.6) $L_0 = L(w_0, E(w_1/w_0), E(q_1/q_0), w_0N_0 + q_0M_0)$

(2.7) $N_1 = N(w_0, E(w_1/w_0), E(q_1/q_0), w_0N_0 + q_0M_0)$.

We have specialized here by assuming for later simplification that only the expected rates of return on inventory holding and money enter the behavioral functions. The consumption and real balance demand functions that are also implied by the maximization process will not be studied further.

The first order conditions for an interior maximum in the first period are:

(2.8') $w_0U_1 - U_2 = 0$

(2.9') $$-U_1 + E\ V_1\left[w_1F_2\ \frac{q_1}{q_0} + \frac{q_1}{q_0}\right] - V_2F_2\ \frac{q_1}{q_0} = 0$$

(2.10') $$-U_1 + E\ V_1\left[w_1F_2\ \frac{w_1}{w_0} + \frac{w_1}{w_0}\right] - V_2F_2\ \frac{w_1}{w_0} = 0$$

With the shorthand notation for the two real returns

$$R_N \equiv w_1/w_0,\ R_M \equiv q_1/q_0$$

we can simplify the first order conditions substantially, using (2.3):

(2.8) $w_0U_1 - U_2 = 0$

(2.9) $U_1 = E\{V_1R_M\}$

(2.10) $U_1 = E\{V_1R_N\}$

The conditions have simple interpretations. The first is the marginal condition for optimal labor supply in period 0. The next two equations are optimal intertemporal allocation conditions. Equation (2.9) is

the standard condition for consumption decisions. And, combining (2.9) and (2.10), we obtain the usual portfolio-allocation condition for a two-asset problem, which equates the marginal-utility-weighted rates of return on the two assets.

We could now proceed to undertake comparative static exercises to derive the properties of the functions (2.6) and (2.7), paying particular attention to the effects of N_0 on labor supply and inventory demand. However, we know in advance that conflicting income and substitution effects will render most derivatives ambiguous. Instead, we ask what are the natural assumptions to make about (2.6) and (2.7).

First consider a change in wealth which, we note, is the only way that N_0 has effects in this simple model. If leisure is a normal good, and if demand for both assets rises when wealth increases, then the natural assumptions are: $L_4 < 0$, $0 < \frac{\partial N_1}{\partial N_0} = w_0 N_4 < 1$. In words, an increase in inventories leads to a reduction in production, and to an increase in inventory carry-over which is less than the increase in the initial inventory holdings. In other words, we are assuming that any increment to wealth will be divided among current consumption, investment in inventories, and investment in money, with positive shares for each.

Next, consider the effects of an increase in w_0, the current relative price, on labor supply (output) and inventory carry-over. From the first argument in (2.6), production will rise if substitution effects dominate. The effects on inventory demand of an increase in w_0, as reflected in the first argument in (2.7), are difficult to predict. Looking at the second argument of each of (2.6) and (2.7), an increase in w_0, given a fixed distribution of w_1, reduces the expected return to inventory holding.

This would be likely to depress inventory demand, and probably current production. The fourth arguments in the labor supply (output) and inventory demand equations are wealth terms; the wealth effects arising from an increase in w_0 would be likely to reduce labor supply and increase inventory demand. The total effects of an increase in w_0 are ambiguous for both behavioral functions. We shall assume that, on balance, high w_0 encourages current production, and reduces inventory demand, as inventories are sold off to take advantage of a currently high relative price.

Finally, we turn to the two rates of return. The effects on labor supply turn on the balance of income and substitution effects, and we assume that the total effect is small. In asset demand functions, we assume that own-return elasticities are positive while cross-elasticities are negative.

For future reference, we summarize the plausible slope assumptions about equations (2.6) and (2.7):

$$\frac{\partial L_0}{\partial w_0} > 0, \ \frac{\partial L_0}{\partial N_0} < 0, \ \frac{\partial L_0}{\partial E(R_N)} \approx 0, \ \frac{\partial L_0}{\partial E(R_M)} \approx 0$$

(2.11)

$$\frac{\partial N_1}{\partial w_0} < 0 \ (?), \ 0 < \frac{\partial N_1}{\partial N_0} < 1, \ \frac{\partial N_1}{\partial E(R_N)} > 0, \ \frac{\partial N_1}{\partial E(R_M)} < 0.$$

Since Y is identical to L, implicit in these functions is a supply function for the yeoman farmer:

$$Y = S(w_0, N_0, \ldots) \tag{2.12}$$

with $\frac{\partial Y}{\partial w_0} > 0$ and $\frac{\partial Y}{\partial N_0} < 0$, and in which there may be rate-of-return effects as well. There is also an inventory-demand function:

$$N = \psi(N_0, E(R_N), E(R_M), \ldots) \tag{2.13}$$

with a possible effect of w_0 as well. The most important implication of the inventory-demand function is that utility maximization implies a type of partial adjustment behavior. That is, if for some reason inventories become too large (small), the individual-firm will not eliminate the excess (shortfall) immediately, but will do so gradually over time since $\frac{\partial N_1}{\partial N_0} < 1$. This is the basic source of the serial correlation of output in the macro model.

Now this yeoman farmer can be placed in a standard Lucas-Phelps world in which there is imperfect information about the current price level, q_0. He will then react in the manner described by equations (2.12) and (2.13) to any disturbance that he believes to be an increase in the relative price of his own good. Apart from real balance effects and adjustments in his nominal money holdings, he will not react to changes in the aggregate price level. Thus, if he is located on a Phelpsian island, he will react to any change in the absolute price of his own output, $W_0 = w_0/q_0$, as if it were partly a relative and partly an absolute price change. That is, his reactions to an increase in observed absolute price in his isolated market will be qualitatively as shown for the reaction to increases in w_0 in equations (2.12) and (2.13).

3. Inventories and the Supply Function of Firms.

In the utility maximization model, we derived a demand for inventories even with perfect competition, a linear production function, and no adjustment costs. This will not be possible in a model of the firm; accordingly we assume a convex cost structure, i.e., increasing marginal production costs. But, for reasons explained more fully in Blinder (1978), this

too turns out not to be sufficient to yield a well-defined inventory policy, and certainly not enough to justify an effect of the inventory stock on production decisions at the micro level.

The nonexistence of a well-defined inventory policy at the level of the competitive firm does not, however, mean that output is independent of inventory stocks at the level of the market, but only that the effects of inventories are indirect: high inventory stocks lead to low prices, and low prices lead to low production. In order to examine conveniently the role of inventories at the firm level, we turn next to a model of a firm with some (at least transitory) monopoly power, that is, with a downward sloping demand curve.[1]

Consider a firm with a demand curve that shifts randomly from period to period:

$$p_t = v_t P_t D(X_t), \tag{3.1}$$

where p_t is the firm's own absolute price, v_t is an identically and independently distributed disturbance in relative price with expectation unity, P_t is the aggregate price level (also random), and $D(X_t)$ is a downward-sloping function of the amount that the firm sells, X_t.[2] We will assume that the firm can observe both v_0 and P_0 before making its current output and sales decisions, while v_t, P_t $(t = 1, 2, \ldots)$ are random variables. Later we shall comment on what happens if the firm cannot distinguish between v_0 and P_0.

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1. Blinder (1978) shows, in a certainty context, that the implications of this model are basically identical to those of a perfect competitor with a sales constraint.

2. D( ) and the other functions introduced below do not have a time index only to economize on notation. Nothing in the nature of the problem requires that D( ) or production costs or inventory holding costs be the same in each period; however, the firm's expected revenues cannot be growing too fast if an optimum is to exist.

Nominal production costs are assumed to be (a) homogeneous of degree one in the absolute price level, (b) a convex function of output, Y_t, and (c) influenced (see below) by unanticipated inflation. Specifically:

$$(3.2) \qquad C_t = P_t c(Y_t, P_t/{}_{t-1}P_t) \qquad c_1 > 0,\ c_{11} > 0,\ c_{12} < 0.$$

Thus marginal costs are assumed to be <u>decreased</u> by unanticipated inflation. The reason is the Friedman-Phelps-Lucas mechanism that we sketched in Section 2: workers do not know the aggregate price level, and hence confuse higher nominal wages with higher real wages. We assume that $\lim_{Y\to 0} c_Y(Y, \ldots) = 0$, and $\lim_{Y\to\infty} c_Y(Y, \ldots) = \infty$, so that the firm will always select an interior maximum for Y_t.

In the current period, the firm must decide how much to produce and how much to sell. These will jointly determine its inventory carry over according to:

$$(3.3) \qquad N_{t+1} = N_t + Y_t - X_t,$$

where N_t is the beginning-of-period inventory stock. N_0 is exogenous. Inventory carrying costs are given by an increasing and convex function, $B(N_t)$.

The firm wants to maximize the expected discounted present value of its real profits. Thus it wants to find:

$$(3.4) \qquad J_0 = \max_{\{X_t,Y_t\}} E_0 \sum_{t=0}^{\infty} \left(\frac{R(X_t,v_t)}{(1+r)^t} - \frac{c(Y_t,U_t)}{(1+r)^t} - \frac{B(N_{t+1})}{(1+r)^{t+1}} \right)$$

where R( ) is the real revenue function,

$$R(X_t,v_t) = p_t X_t/P_t = v_t D(X_t) X_t,$$

which we assume has the following properties:

$$R_X \geq 0,\ R_{XX} < 0,\ \lim_{X \to 0} R_X(X,v) = +\infty,\ D(X_t)X_t \text{ is bounded above}$$

The latter assumptions assure us that X_t will always achieve an interior maximum. To simplify the notation, we introduce the symbol $U_t = P_t / {}_{t-1}P_t$ for unanticipated inflation.

The problem is set up in dynamic programming form by defining:

$$\pi_t = R(X_t, v_t) - c(Y_t, U_t) - \frac{B(N_{t+1})}{1+r}$$

$$J_1 = \max_{\{X_t, Y_t\}} E_1 \sum_{t=1}^{\infty} \left[\frac{R(X_t, v_t)}{(1+r)^{t-1}} - \frac{c(Y_t, U_t)}{(1+r)^{t-1}} - \frac{B(N_{t+1})}{(1+r)^t} \right],$$

so that (3.4) may be rewritten:

$$(3.5) \qquad J_0 = \max_{\{X_0, Y_0\}} \pi_0 + \frac{J_1}{1+r}.$$

It is clear from the set-up of the problem that the J_t functions depend on the initial inventory stock, N_t; the initial realizations of the two random variables, v_t and P_t; the joint distribution of all the stochastic variables; and the functional forms of all the D, c, and B functions. Since only the inventory stock is a control variable of the firm, we shall simply write $J_t = J_t(N_t)$. Our assumptions imply that the $\pi_t(\ )$ are concave, continuous, bounded functions; accordingly the J_t are concave and continuous and, given $r > 0$ and the assumed stationarity of v_t, bounded; an optimal policy therefore exists.

To solve the problem for the first-period solution, it is easiest to use (3.3) to eliminate Y_0, and treat X_0 and N_1 as the firm's decision variables. First order conditions for an interior maximum, on which we concentrate, are then:

$$(3.6) \qquad R_X(X_0, v_0) - c_Y(X_0 + N_1 - N_0, U_0) = 0$$

$$(3.7) \qquad \frac{J_1'(N_1) - B'(N_1)}{1+r} - c_Y(X_0 + N_1 - N_0, U_0) = 0$$

The first condition equates marginal revenue with marginal cost as usual. The second says that the marginal value of adding one unit to inventories must be equal to the sum of the costs of producing that unit and carrying it over to the next period. Henceforth, to simplify the notation, we will work with the composite function:

$$G(N_1) = J_1'(N_1) - B'(N_1).$$

Since $J_1'' < 0$ and $B'' > 0$, $G'(N_1) < 0$.

These first order conditions imply optimal decision rules for current sales and inventory-carry over, and therefore production, of the form:

$$(3.8) \qquad X_0 = X(N_0, v_0, U_0, 1+r)$$

$$(3.9) \qquad N_1 = N(N_0, v_0, U_0, 1+r)$$

$$(3.10) \qquad Y_0 = S(N_0, v_0, U_0, 1+r) = X(.) + N(.) - N_0.$$

The derivatives of these functions can be worked out by the usual comparative statics technique. The following summarizes their relevant properties:

$$0 < \frac{\partial X_0}{\partial N_0} < 1, \ \frac{\partial X_0}{\partial v_0} > 0, \ \frac{\partial X_0}{\partial U_0} > 0, \ \frac{\partial X_0}{\partial(1+r)} > 0$$

$$(3.11) \qquad 0 < \frac{\partial N_1}{\partial N_0} < 1, \ \frac{\partial N_1}{\partial v_0} < 0, \ \frac{\partial N_1}{\partial U_0} > 0, \ \frac{\partial N_1}{\partial(1+r)} < 0$$

$$-1 < \frac{\partial Y_1}{\partial N_0} < 0, \ \frac{\partial Y_0}{\partial v_0} > 0, \ \frac{\partial Y_0}{\partial U_0} > 0, \ \frac{\partial Y_0}{\partial(1+r)} < 0$$

We are most interested in the effects of the initial stock of inventories. An increase in N_0 leads to a drop in current production, an increase in current sales (i.e., a cut in relative price), and an increase in next period's inventories, but by less than the increase in current inventories. Thus the apparent partial adjustment feature is maintained, though it has nothing to do with the usual adjustment cost derivations of stock adjustment models.

Turning next to the relative price shock (shift in the demand curve), the profit maximizing firm will respond by raising both sales and output. But the sales response is greater so that inventory carry-over falls. If instead there is an absolute price level shock, i.e., unanticipated inflation, with no change in the firm's relative price, it will again respond by raising both sales and production. But this time production responds more, so that inventory carry over is greater; this is a result of the lower current production costs.

The intuition behind these results is straightforward once we keep in mind that the firm is operating on two margins: it is deciding how much to produce for inventories, and it is deciding how much to withdraw from inventories for sale. When the firm's <u>relative</u> price increases, the rewards for selling today (rather than tomorrow) are increased. But neither production costs nor the rewards for selling tomorrow (assuming that v_1 and v_0 are independent) are affected. So the incentive to raise sales is greater than the incentive to raise output. Inventory stocks get depleted. By contrast, a shock to the absolute price level has no effect on the firm's decision to sell today rather than tomorrow, but it does reduce costs (assuming $c_{12} < 0$). So production for sale today or in any future period looks more attractive than it did before the shock, but the effect on production is clearly greater than it is on sales.

Finally, let us consider what may happen if the firm, like the workers, cannot distinguish between v_0 and U_0. At first blush, it would appear that we would get some sort of weighted-average reaction, with the weights dependent on the extent to which the firm thinks the shock is real (v_0) versus nominal (U_0). Sales and output rise, but we cannot predict which will rise more. But this is too hasty. The basic mechanism by which unanticipated inflation reduces costs is that firms know the aggregate price level while workers do not. If this informational asymmetry disappears, U_0 should be dropped from the cost function. With $c_{12} = 0$, the firm would no longer react to unanticipated changes in the price level, so that reactions to shocks that were <u>either</u> v_0 or U_0 would be weighted averages of the reactions to relative-price shocks and zero. That is, they would be muted versions of the reaction of the firm to a relative demand shift, precisely as in the yeoman farmer case.

To summarize, then, the models of a yeoman farmer and of a monopolist have almost identical predictions. Both current production and inventory carry over depend on unanticipated inflation (interpreted as an increase in relative price), on the current relative price, on the initial stock of inventories, and on expected rates of return.

4. Inventories and Persistence Effects

In this section we embody the results of the previous two sections about the effects of inventories on supply, and the determinants of the demand for inventories, in the simplest possible macro model. We show that the inclusion of inventories produces serial correlation of output, but that despite this serial correlation there is no room in the model for systematic monetary policy to affect the behavior of output.

We start with aggregate output. The key insight of the previous two sections was that a high level of current inventories reduces current output. In particular, (3.11) implies that an increase in current inventories decreases current output, but by less than the full amount of the increase in inventories. We know also that an unanticipated increase in the current price level leads to increases in aggregate output. We therefore write the aggregate output equation in the form

$$(4.1) \qquad Y_t = K_t + \gamma(P_t - {}_{t-1}P_t) + \lambda(N^* - N_t) + e_{1t}, \ \gamma > 0, \ 0 < \lambda < 1$$

Here Y_t and N_t are the levels of output and inventories respectively, and P_t is the log of the price level, with ${}_{t-1}P_t$ the expectation taken at time t-1 of the log of the price level at t.[1] K_t is a term representing potential output and N* represents desired inventories, taken to be constant over time in this simple model. The apparent specialization in (4.1) to a stock-adjustment version of inventory behavior, through the inclusion of the term N*, is entirely innocuous given the linearity of the equation, since the term λN^* is constant and could be embodied in the K_t term without affecting the behavior of the model. The micro theory does imply that $\partial Y_t/\partial N_t > -1$, and we are merely assuming this derivative is constant. Finally, e_{1t} is a serially uncorrelated disturbance term with expectation zero.

The demand for inventories was seen in the earlier sections to be an increasing function of the current level of inventories, with an increase in current inventories increasing desired inventories less than one-for-one. We saw also that unanticipated inflation probably causes inventories to be drawn down. We summarize these two essential elements in

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1. The mixed log-level assumptions are made purely for convenience.

$$(4.2) \qquad N_{t+1} - N_t = \theta(N^* - N_t) - \phi(P_t - {}_{t-1}P_t) + e_{2t}. \quad 0 < \theta < 1, \ \phi \geq 0$$

Once again, the use of a stock adjustment form for (4.2) is inessential.

It is worth noting here that $\theta > \lambda$. The reason is that the model of inventory behavior in Section 3 implies (see eqs. 3.11) that $\frac{\partial X}{\partial N_0} > 0$. Since $\frac{\partial X}{\partial N_0} = 1 + \frac{\partial Y}{\partial N_0} - \frac{\partial N_1}{\partial N_0}$, this implies--in the notation of (4.1) and (4.2)--that $1 + (-\lambda) - (1 - \theta) = \theta - \lambda$ is positive. The disturbance term e_{2t} is white noise.

To provide intuitive understanding of (4.1) and (4.2), consider a situation in which inventories exceed desired inventories. In the absence of price surprises, firms will be running off their inventories slowly, selling more than they produce, and producing less than they would if inventories were at their desired level. This adjustment pattern would continue smoothly, unless there were unanticipated changes in the general price level. If there is an unanticipated change in the general price level, firms increase sales, raising both production and sales out of inventory to do so. Hence a positive price surprise this period implies a lower stock of inventories next period.

To close the model it is now necessary only to add an aggregate demand equation. In keeping with our aim of maximum simplicity, we append an interest inelastic demand function for money:

$$(4.3) \qquad M_t = P_t + aX_t + e_{3t}, \ a > 0.$$

Here M_t is the log of the money stock.[1] The random variable e_{3t} is again white noise. In writing (4.3), we have to decide whether it is the level of output (Y_t), or the level of sales (X_t) that should represent

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1. See the previous footnote.

the volume of transactions. We see no convincing basis for selecting one rather than the other; fortunately, most of the results we will obtain are independent of the choice of scale variable in (4.3).[1]

In order to demonstrate that (i) output is serially correlated and (ii) systematic monetary policy has no effects on output in the current system so long as expectations are formed rationally, it is not necessary to solve explicitly for the rational expectations solution of the model. To make that clear, we shall start by deriving an expression for real output as a function only of current and past unanticipated inflation, and the disturbances.

First, from (4.2), the level of inventories is seen to be a function only of past unanticipated inflation, and the stochastic term in the inventory demand function, e_{2t}. We denote the unanticipated increase in the price level by u_t:

$$u_t = P_t - {}_{t-1}P_t. \tag{4.4}$$

Then, solving the difference equation (4.2), and assuming the economy has an infinite past so that initial conditions can be ignored (given that the model is stable), we have

$$N_t = N^* - \phi \sum_0^{\infty} (1 - \theta)^i u_{t-1-i} + \sum_0^{\infty} (1 - \theta)^i e_{2t-1-i} \tag{4.5}$$

Equation (4.5) repeats what we already know--that an unanticipated increase in the price level leads inventories to be drawn down, and then only gradually built back to their original level, so that the effect of any burst of unanticipated inflation on the current stock of inventories is smaller the further in the past the inflation surprise occurred.

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1. But see footnote 1 on p. 26 below.

Now we can substitute (4.5) into (4.1) to obtain the level of output:

$$(4.6) \qquad Y_t = K_t + \gamma u_t + \lambda\phi \sum_0^\infty (1 - \theta)^i u_{t-1-i} + e_{1t} - \lambda \sum_0^\infty (1 - \theta)^i e_{2t-1-i}.$$

Equation (4.6) shows that output is positively serially correlated, since unanticipated inflation in the current period leads to higher output in the current period and in all subsequent periods. Depending on the relative magnitudes of γ and $\lambda\phi$, unanticipated inflation may have its maximal effect on output in the period it occurs, or one period later, and thereafter the effects decline geometrically. If unanticipated inflation has a small direct effect on output, so that γ is small, but leads to a large reduction in inventories, so that ϕ is large, then the inventory rebuilding effects of unanticipated inflation on output will predominate, and the maximum impact on output will occur in the period following a given unanticipated increase in the price level.

Figure 4.1 shows how the stock of inventories and level of output are affected by unanticipated inflation. First, unanticipated inflation reduces the stock of inventories, as sales are increased in response to what the firm regards in part as an increase in the relative price of output. Inventories are gradually built back up; the $(1 - \theta)^i$ terms in (4.5) result from the partial adjustment of inventories. Equation (4.6) shows that output is increased by current unanticipated inflation, through an increase in labor supply. Then in subsequent periods output is higher than it would otherwise have been, as a result of the need to rebuild depleted inventories.

Equation (4.6) also shows that systematic monetary feedback rules have no impact on the behavior of output under rational expectations.

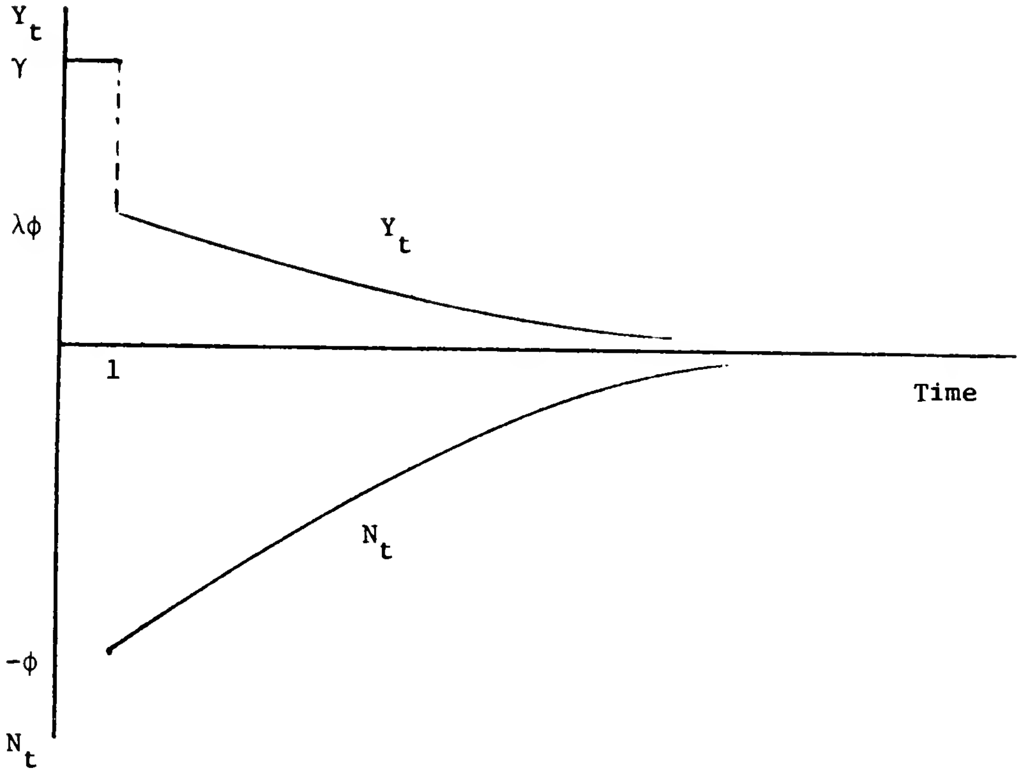

Figure 4.1: Dynamic adjustment of output and inventories to an unanticipated increase in the money stock.

Output is affected only by stochastic disturbances and unanticipated inflation. While systematic feedback rules can produce anticipated inflation, they cannot produce unanticipated inflation, if the feedback rule depends only on information that is available at the time expectations are formed and expectations are rational, as we assume to be the case.

For completeness, we examine also the determinants of the current price level. We note first that

(4.7) $$N_{t+1} - N_t = Y_t - X_t.$$

Combining (4.1), (4.2), (4.3) and (4.7), we obtain

(4.8) $$P_t = M_t - a[K_t + (\phi + \gamma)u_t + (\lambda - \theta)(N^* - N_t) + e_{1t} - e_{2t}] - e_{3t}.$$

The price level is accordingly proportional to perfectly anticipated increases in the money stock, and is a decreasing function of the stock of inventories.[1]

In concluding this section, it is worthwhile emphasizing once more the basic source for the serial correlation of output. An unanticipated increase in the price level in this model leads firms to sell out of inventories at the same time as they increase production to take advantage of what is (incorrectly) perceived as an increase in relative price. Then in subsequent periods production remains high as stocks of goods are rebuilt. The serial correlation of output does not, however, imply that anticipated monetary policy has real effects.

5. Inventories and Monetary Policy.

The serial correlation of output demonstrated in the simple model of the preceding section did not lead to a potential role for pre-announced monetary policy in affecting the behavior of output. In this section we show that once the demand for inventories is made a function of the real interest rate, then pre-announced monetary policy may indeed have real effects on the behavior of the economy. The source of the effectiveness of monetary policy is that a monetary policy which affects the expected rate of inflation also changes the real rate of interest, thus the demand for inventories, and thus production.

The slightly extended version of the model of Section 4 with which we work in this section is:

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1. We noted earlier that our results were not essentially dependent on whether sales or output entered the money demand function (4.3). There is however one factor suggesting that sales are the more appropriate argument. Given that output is a decreasing function of the stock of inventories, the current version of (4.3) implies that inventories exert a deflationary effect on the price level. If output instead of sales were the scale variable in (4.3), large inventories would tend to increase the price level, by reducing the level of output and thus money demand.

$$(5.1)\qquad Y_t = K + \gamma(P_t - {}_{t-1}P_t) + \lambda(N^*_{t+1} - N_t) + e_{1t}$$

$$(5.2)\qquad N_{t+1} - N_t = \theta(N^*_{t+1} - N_t) - \phi(P_t - {}_{t-1}P_t) + e_{2t}, \ \lambda < \theta$$

$$(5.3)\qquad N^*_{t+1} = N^* - br_t$$

$$(5.4)\qquad M_t - P_t = a_1X_t - a_2i_t + e_{3t}$$

$$(5.5)\qquad D_t = c_1Y_t + c_2(M_t - P_t) - c_3r_t + e_{4t}$$

$$(5.6)\qquad X_t = Y_t + N_t - N_{t+1}$$

$$(5.7)\qquad X_t = D_t$$

$$(5.8)\qquad i_t = r_t + {}_tP_{t+1} - P_t$$

The changes from the model of Section 4 are obvious. First, desired inventories in equation (5.3) are now a decreasing function of the real interest rate, instead of being assumed constant. Second, the demand for money now becomes a function of the nominal interest rate. Equation (5.5) gives the demand for goods, and (5.6) gives the level of sales. Equation (5.7) states that the goods market clears and (5.8) defines the real interest rate.[1]

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1. In using this definition for the real interest rate, we depart slightly from the Phelpsian island paradigm. The island paradigm does not allow individuals to know the current aggregate price level with certainty, while equation (5.8) assumes they do know the current price level. A relatively simple way of avoiding this difficulty would appear to be to define anticipated inflation as ${}_{t-1}P_{t+1} - {}_{t-1}P_t$, a device adopted by Sargent and Wallace (1975). However, this too is inconsistent with the island story, since the absolute price in each island gives each individual some information about the current price level. We should actually write ${}_tP_t$ instead of P_t in (5.8), where ${}_tP_t$ is the current estimate of the price level conditional on information available currently. We know that ${}_tP_t$ is a weighted average of the actual aggregate price level and the expectation of P_t conditional on knowledge of the aggregate price level and all other history up to and including t-1. Thus any effects captured in the present version would be present in the more accurate--and considerably more difficult--consistent island paradigm, so long as knowledge of the current nominal interest rate does not serve to identify the current aggregate price level--as it does not, in the present model, in which the money demand and other disturbances prevent identification. Since, in this section, we deal only with perfectly anticipated inflation, the issue discussed in this footnote does not affect the results obtained here.

Working with the model set out above in full generality involves the solution of a complicated rational expectations equation for the price level. Since we are, in this section, concerned only with the effects of anticipated changes in the money stock, we do not work with the full model. Instead, we examine the effects of perfectly anticipated monetary changes[1] by setting all stochastic terms and the unanticipated inflation term $(P_t - {}_{t-1}P_t)$ identically to zero.

Using the notation L for the lag operator, and omitting constants, it can be shown that the level of inventories, N_t, the level of output, Y_t, and the real interest rate r_t, respectively, are given by:

$$N_t = N^* - \frac{\theta b L r_t}{1 - (1-\theta)L} \tag{5.9}$$

$$Y_t = - \frac{\lambda b(1-L)}{1 - (1-\theta)L} r_t \tag{5.10}$$

$$r_t = - \frac{c_2 a_2 ({}_tP_{t+1} - P_t)}{c_3 + c_2 a_2 + \dfrac{[\theta(1 - c_2 a_1) + \lambda(c_1 - 1 + c_2 a_1)]b(1-L)}{1 - (1-\theta)L}} \tag{5.11}$$

Examining (5.11), we see the basic source of the nonneutrality of anticipated money in this model. Anticipated inflation reduces the real rate of interest,[2] and anticipated inflation obviously is affected by the growth of the money stock. Further, by looking at (5.11) we see that there are two necessary conditions for the nonneutrality of money in this model, namely that both c_2 and a_2 be nonzero. The parameter c_2 reflects the role of the real balance effect in the goods market, and a_2 reflects the interest elasticity of the demand for money. Equation

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1. By "perfectly anticipated" we mean that the monetary changes being discussed have always been known about; for a more precise definition see Fischer (1977b).

2. This statement assumes that the denominator is positive. Since $\theta > \lambda$, a sufficient condition is $c_1 + c_2 a_1 < 1$, an assumption we make.

(5.9) shows that the stock of inventories is negatively related to the current and past real rates of interest and (5.10) shows that the level of output is related to the rate of change of the real interest rate. The coefficient b that appears in (5.9) and (5.10) is likely to be small.

The remaining task is to study the determination of the price level and the rate of inflation. Working with the model (5.1) through (5.8), with the stochastic terms set to zero, we obtain the equation for the price level:

$$(5.12) \qquad P_t = b_0 + b_1 P_{t-1} + b_2\, {}_tP_{t+1} + b_3\, {}_{t-1}P_t + b_4 M_t + b_5 M_{t-1}$$

with $b_1, b_2, b_4 > 0$ and $b_3, b_5 < 0$

$$\sum_1^5 b_i = 1, \quad b_2 + b_4 = 1, \quad b_1 + b_3 + b_5 = 0$$

The coefficients b_1 through b_5 are defined in the appendix; none of these b_i should be confused with the parameter b in equation (5.3). The form in which (5.12) is written emphasizes that the current price level P_t is a function of the anticipated price level, ${}_tP_{t+1}$, as well as of the lagged price level, lagged expectations, and current and lagged money stocks. Since we are working with the assumptions that all stochastic terms are zero and that there is perfect foresight about the behavior of the money stock, the expectations in (5.12) will be equal to the actual values of the price level.

As shown by Blanchard (1978), there are a variety of solutions for equations of the form of (5.12), some of which make the price level a function only of lagged money stocks. We choose to work with a solution that makes the current price level a function of both lagged and future money stocks, since we believe it reasonable that individuals in forming

their expectations of future price levels will take into account the expected evolution of the money stock. The general form of a solution which takes both lagged and future behavior of the money stock into account is studied in some detail in Fischer (1977b). A particular form of the solution that applies when there is no uncertainty is:

$$(5.13) \qquad P_t = \delta + \sum_{0}^{\infty} \pi_i M_{t+i} + \theta M_{t-1} + \mu P_{t-1}$$

where μ is the solution that is less than unity to:

$$(5.14) \qquad b_2\mu^2 + (b_3 - 1)\mu + b_1 = 0$$

and

$$(5.15) \qquad \pi_0 = \left[1 - \frac{b_2\mu}{b_1}\right]\left[\frac{\mu - b_2\mu^2}{b_1}\right] > 0$$

$$\pi_i = \left(\frac{b_2\mu}{b_1}\right)^i \pi_0 \qquad i = 1, 2, 3, \ldots.$$

$$\theta = \frac{b_5\mu}{b_1} < 0$$

and where

$$0 < \frac{b_2\mu}{b_1} < 1$$

Note also that

$$\sum_{0}^{\infty} \pi_i = \frac{\mu}{b_1}[1 - b_2\mu]$$

$$\theta + \sum_{0}^{\infty} \pi_i = 1 - \mu$$

Equipped with the rational expectations solution for the price level, (5.13), and equations (5.9) through (5.11), we can now study the effects of monetary changes on the economy. In particular, we shall first discuss the effects of a perfectly anticipated once over change in the level of the money stock, and then discuss the effects of a once over change in the growth rate of the money stock. In each case we shall assume that the change takes place in period τ, and that it has always been anticipated that the change would occur.

The effects of a permanent change in the stock of money on the price level in previous and subsequent periods are described in Fischer (1977b). Figure 5.1 shows the dynamic adjustment of the (log of the) price level to a 1 percent change in the money stock that occurs in period τ. The rate of inflation $(P_t - P_{t-1})$ accelerates up to period τ, and thereafter slows down. Figure 5.1 shows also the implied behavior of the real interest rate, r_t, which falls as the inflation rate accelerates up to time τ, and then starts rising as the inflation rate slows down.

The corresponding behavior of the level of inventories and the level of output are shown in Figure 5.2. Inventories build up as the real interest rate falls, and then, after the increase in the money stock, start being worked off. The behavior of output can be understood by combining (5.1) and (5.2), with all stochastic terms set to zero:

$$Y_t = K_t + \frac{\lambda}{\theta} (N_{t+1} - N_t)$$

The rate of production is related to the rate of change of inventories. Accordingly, output is increasing up to the period before the money stock changes; thereafter output actually decreases below its steady state value as the inventory excess is worked off. In the longest of runs, the once-

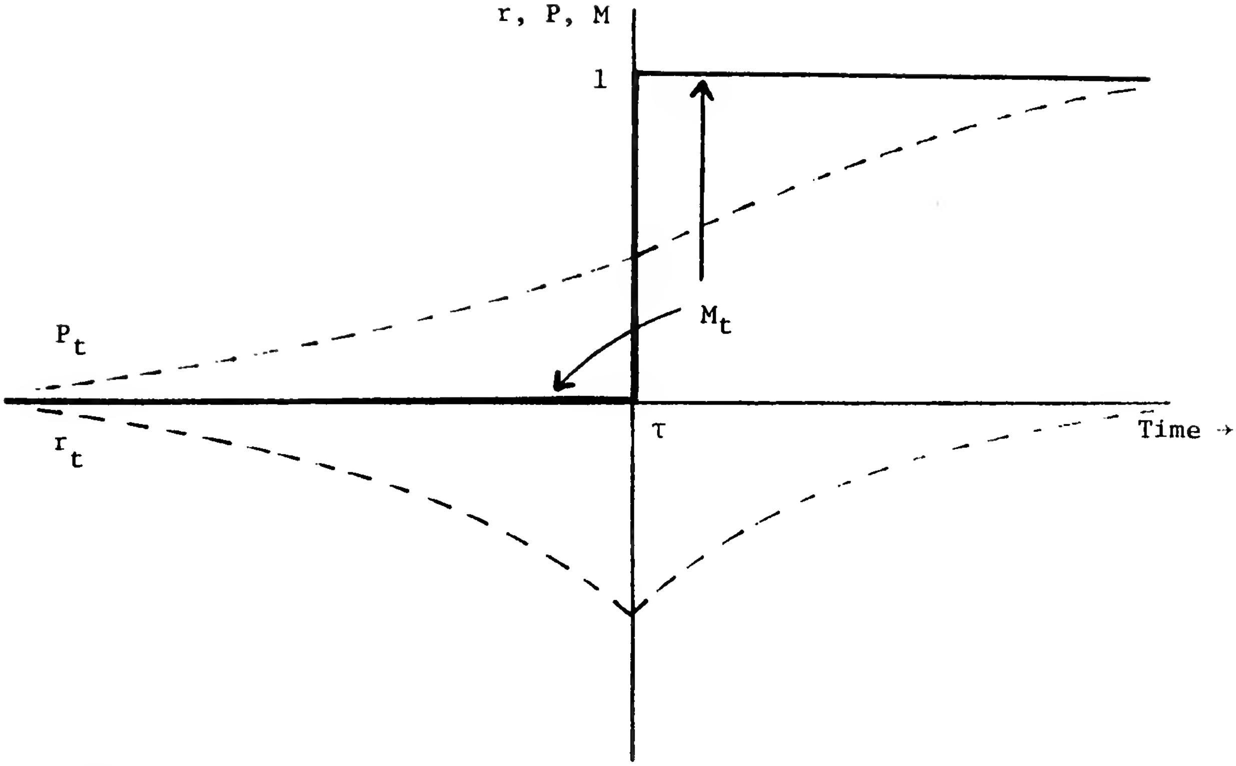

Figure 5.1: Dynamic adjustment of the price level and real interest rate to a permanent change in the money stock.

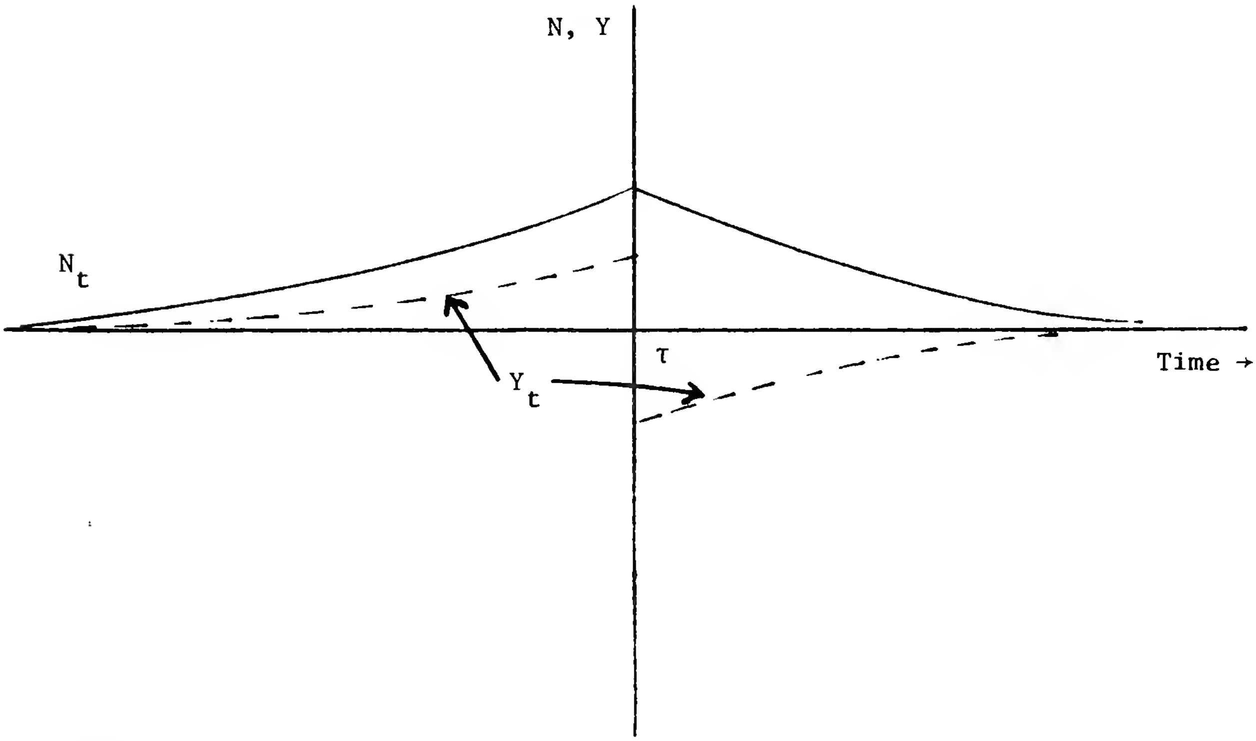

Figure 5.2: Dynamic adjustment of the stock of inventories and level of output to a permanent change in the money stock.

over change in the money stock is neutral, resulting only in a proportionately higher price level, but the real economy is affected by the anticipation of the change in the money stock, and continues to be affected after the change has taken place.

We turn our attention next to the effects of a permanent change in the growth rate of the money stock. Before we look at the details, it is worth thinking through the consequences of such a change. Ultimately, we expect the rate of inflation to be equal to the growth rate of money. Looking now at (5.11), we see that the real interest rate is reduced by increases in the expected inflation rate, and we should therefore expect a permanent increase in the growth rate of money to reduce the steady state real interest rate. Equation (5.9) shows that, with the new higher rate of growth of money, the level of inventories in the steady state will be higher. From (5.10), however, we note that the level of output is affected only by the first difference of the real interest rate. Therefore, in the steady state, the level of output will be unaffected by the change in the growth rate of money.

Once more, the key to understanding the dynamic adjustment of the economy to the monetary change is the behavior of the price level. This time, we plot the rate of inflation, rather than the price level, in Figure 5.3. The inflation rate increases over the entire period; it accelerates up to the time the growth rate of the money stock changes (between periods τ and $\tau+1$), and then decelerates after the change in the growth rate of money.[1] Precise details are provided in the appendix. Given the continuously increasing rate of inflation, the real rate of interest falls continuously.

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1. Note that the overshooting of the inflation rate above the growth rate of money occurs <u>before</u> there is any change in monetary growth.

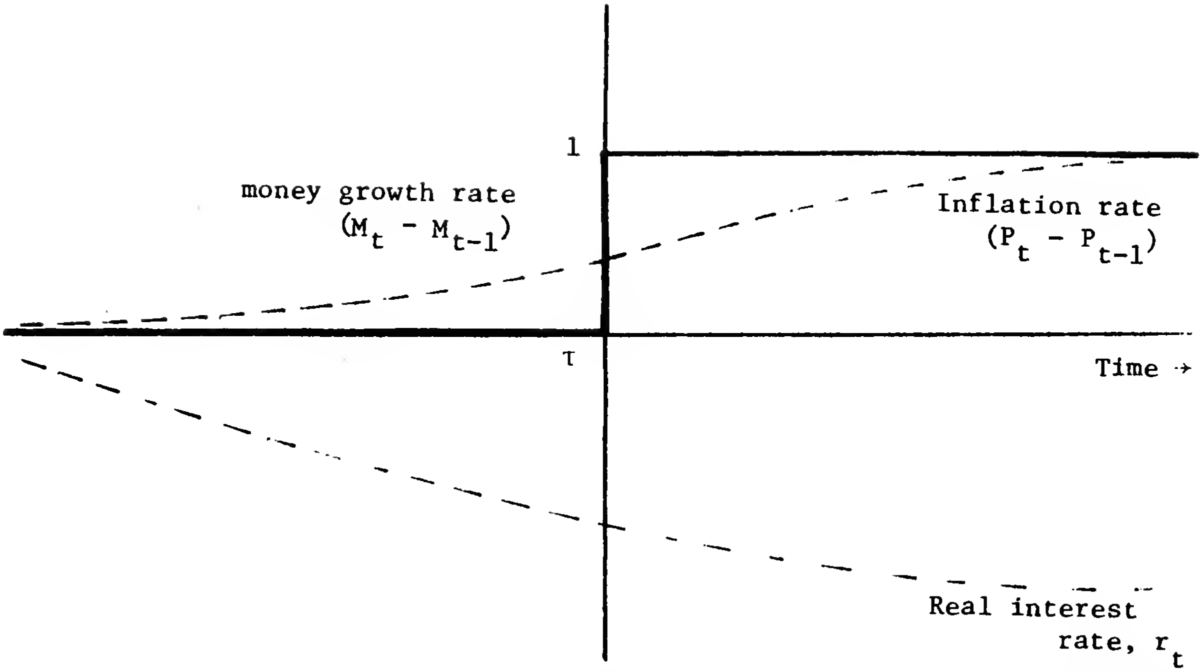

Figure 5.3: Dynamic adjustment of the rate of inflation and real interest rate to an increase in the growth rate of the money stock.

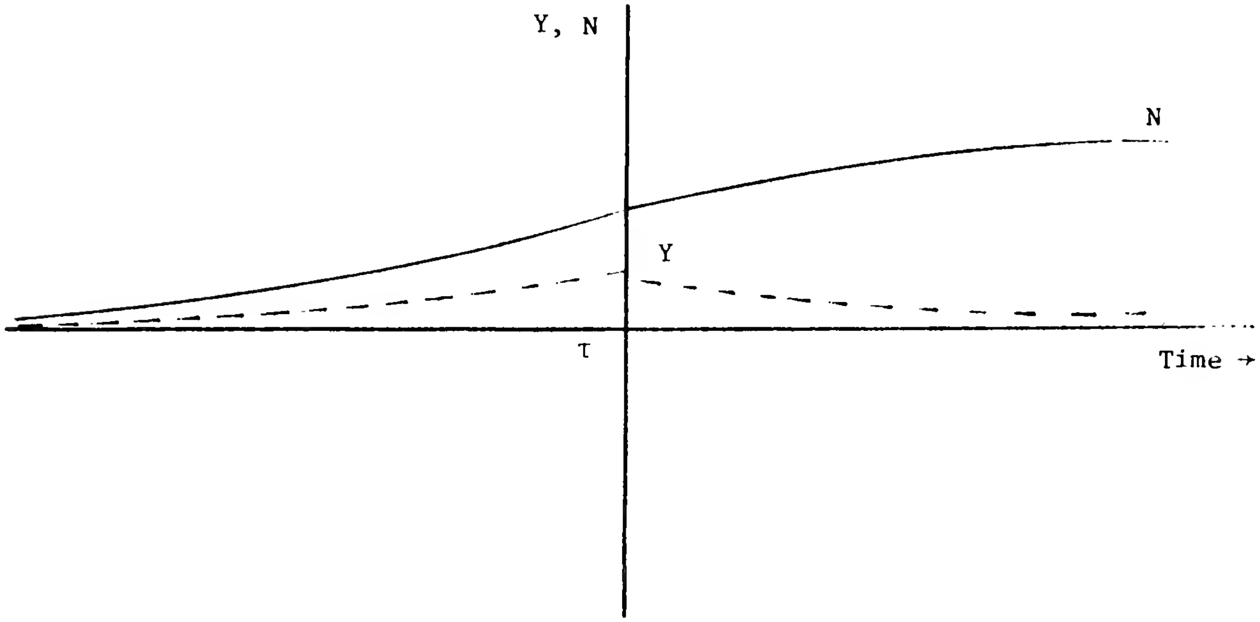

Figure 5.4: Dynamic adjustment of the stock of inventories and level of output to an increase in the growth rate of the money stock.

Figure 5.4 shows the behavior of inventories and output. The stock of inventories builds up steadily to its new higher level, but the rate of increase of inventories is highest between periods τ and $\tau-1$; thereafter the rate of increase of inventories slows down. Accordingly, the level of output is at a maximum in period $\tau-1$, and gradually slows down thereafter. The change in the growth rate of money has its maximal effect on output in the period before the change, but continues to affect output behavior thereafter. Only asymptotically does output return to its steady state level.

To sum up, the inclusion of the real interest rate in the demand function for inventories, coupled with the real balance effect on the demand for goods, provides a potential route through which anticipated monetary policy can affect the behavior of output. The behavior of output depends on the change in inventories. In response to a permanent change in the stock of money, inventories build up in anticipation of the change in the money stock, and are then worked off after the change occurs. The proximate cause of the inventory changes in this case is the behavior of the real interest rate, which is in turn fundamentally determined by the expected rate of inflation. Similarly, the response to a permanent increase in the rate of growth of growth of the money stock, which permanently reduces the real rate of interest, is that inventories are built up slowly to a new permanently higher level. Output correspondingly increases above its steady state level, being at its highest level in the period before the growth rate of money changes, and thereafter slowly returns to its steady state level.

6. Conclusions.

This paper has examined the way in which the inclusion of storable output modifies the aggregate supply function that is normally used in rational expectations models. The microeconomic foundations examined in Sections 2 and 3 led to a type of "partial adjustment" mechanism for inventories, in which excess inventories are worked off only slowly over time, rather than all in one period. They are worked off in part by reducing the level of output.

Including this sort of inventory behavior changes the dynamics of the macro model substantially. In particular, in a simple rational-expectations model in which output disturbances are otherwise serially uncorrelated, inventory adjustments lead to "business cycles," that is, to long-lived effects on output. In particular, unanticipated changes in the money stock simultaneously increase current output and decrease inventories, as some inventories are sold off to meet the higher demand. Then, in subsequent periods, output is raised to restore the depleted inventories. This mechanism, which we examined in Section 4, is the most important of this paper in that it provides a very natural vehicle for the propagation of business cycles. A look at the data suggests that this vehicle is probably of great empirical importance.

Finally, in Section 5, we examined the effects of perfectly anticipated changes in the money stock on output in a model in which desired inventory stocks are a function of the real interest rate. In that case, since a permanent change in the stock of money, while ultimately neutral, alters the time path of the real interest rate, it also alters the paths of inventories and output. In particular, inventories and output are raised in anticipation of the change; and a long period of reduced output

follows the monetary change, as the excess inventories are worked off. A permanent increase in the <u>growth rate</u> of money leads to a permanent increase in the stock of inventories, and to an output level that remains above the steady state level both before and after the change in the growth rate of money, as inventories are accumulated.

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Appendix

1. In this appendix we briefly indicate some of the calculations underlying statements in Section 5 of the paper. First, the values of the coefficients b_1 through b_5 in (5.12) are:

$$b_1 = \xi[(1-\theta)(c_3 + c_2a_2 + a_2c_3) + (1 + a_2)\beta b + a_1a_2c_2(\theta-\lambda)b] < 0$$

$$b_2 = \xi a_2[c_3 + \beta b + c_2a_1(\theta-\lambda)b] < 0$$

$$b_3 = -\xi a_2[(1-\theta)c_3 + \beta b + c_2a_1(\theta-\lambda)b] < 0$$

$$b_4 = \xi[c_3 + c_2a_2 + \beta b] > 0$$

$$b_5 = -\xi[(1-\theta)(c_3 + c_2a_2) + \beta b] < 0$$

$$\beta \equiv \theta(1 - c_2a_1) + \lambda(c_1 - 1 + c_2a_1) > 0 \quad \text{since } \theta > \lambda$$

2. The effects of a fully-anticipated permanent change in the stock of money on the price level are calculated in Fischer (1977b). It is shown there (equation (28)) that in period t, in which the money supply changes:

$$\frac{\partial P_t}{\partial M_t} = \frac{\mu(1 - b_2\mu)}{b_1 - b_2\mu^2} \leq 1$$

In earlier periods:

$$\frac{\partial P_{t-i}}{\partial M_t} = \left(\frac{b_2\mu}{b_1}\right)^i \frac{\partial P_t}{\partial M_t}$$

Thus, up to period t, the inflation rate is given by:

$$\text{(A2.1)} \qquad \frac{\partial P_{t-i}}{\partial M_t} - \frac{\partial P_{t-i-1}}{\partial M_t} = \left(\frac{b_2\mu}{b_1}\right)^i \left(1 - \frac{b_2\mu}{b_1}\right) \frac{\partial P_t}{\partial M_t}, \quad i = 0, 1, \ldots$$

The inflation rate therefore increases up to period t.

In subsequent periods

$$\frac{\partial P_{t+1}}{\partial M_t} = 1 - \frac{\mu^i(b_1 - \mu)}{b_1 - b_2\mu^2} > 0 \qquad i = 0, 1, 2, ..$$

Thus the inflation rate is

$$\text{(A2.2)} \qquad \frac{\partial P_{t+i}}{\partial M_t} - \frac{\partial P_{t+i-1}}{\partial M_t} = \frac{\mu^{i-1}(b_1 - \mu)(1-\mu)}{b_1 - b_2\mu^2}$$

The inflation rate therefore decreases after period t+1.[1]

Finally, we want to show that the maximum inflation rate occurs between periods (t-1) and t. We accordingly have to show that

$$\frac{\partial P_t}{\partial M_t} - \frac{\partial P_{t-1}}{\partial M_t} > \frac{\partial P_{t+1}}{\partial M_t} - \frac{\partial P_t}{\partial M_t}$$

or
$$\left(1 - \frac{b_2\mu}{b_1}\right)\left(\mu(1 - b_2\mu)\right) > (b_1 - \mu)(1-\mu)$$

or
$$\frac{\mu}{b_1}\,\frac{b_1 - b_2\mu}{b_1 - \mu}\,\frac{1 - b_2\mu}{1-\mu} > 1$$

Since $b_2 < 1$, and $\mu < b_1$ (by the assumption noted in the preceding footnote), it will suffice to show that

$$\frac{\mu(1 - b_2\mu)}{b_1(1-\mu)} > 1,$$

or
$$\mu(1 - b_2\mu) > b_1(1-\mu), \text{ or } \mu - b_2\mu^2 - b_1 + b_1\mu > 0$$

Now, from (5.14), we can substitute for $-(b_1 + b_2\mu^2)$, so we have to show:

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1. This statement requires $b_1 > \mu$, which is guaranteed if $[a_2(\theta-\lambda)(1-c_1-c_2a_1) + a_2\theta c_1 - c_3a_1\lambda - c_2a_1a_2\lambda] > 0$, a condition we assume. It is satisfied if θ is sufficiently greater than λ, for instance.

$$\mu + (b_3 - 1)\mu + b_1\mu > 0$$

or $$(b_1 + b_3)\mu > 0.$$

Since $b_1 + b_3 = -b_5 > 0$, the inflation rate has been shown to be at a maximum between periods (t-1) and t.

3. To derive the behavior of N_t, r_t and Y_t, we work from (5.9) and (5.11) to obtain

(A3.1) $$N_t - N^* = \sum_1^{\infty} \Psi_i ({}_{t-i}P_{t-i+1} - P_{t-i})$$

$$\Psi_1 = \frac{c_2 a_2 \theta b}{c_3 + c_2 a_2 + \beta b}$$

$$\Psi_i = \left[\frac{(c_3 + c_2 a_2)(1-\theta) + \beta b}{c_3 + c_2 a_2 + \beta b}\right]^{i-1} \Psi_1$$

$$= \left(\frac{-b_5}{b_4}\right)^{i-1} \Psi_1$$

(A3.2) $$r_t = \frac{-c_2 a_2}{c_3 + c_2 a_2 + \beta b} \sum_{i=0}^{\infty} \xi_i ({}_{t-i}P_{t+1-i} - P_{t-i})$$

$$\xi_0 = 1$$

$$\xi_1 = \frac{\theta \beta b}{c_3 + c_2 a_2 + \beta b}$$

$$\xi_i = \left[\frac{(c_3 + c_2 a_2)(1-\theta) + \beta b}{c_3 + c_2 a_2 + \beta b}\right]^{i-1} \xi_1, \quad i = 2, \ldots, \infty$$

$$= \left(\frac{-b_5}{b_4}\right)^{i-1} \xi_1$$

We also use

$$Y_t = \frac{\lambda}{\theta}(N_{t+1} - N_t) \tag{A3.3}$$

4. We do not intend giving formulae corresponding to all the figures in Section 5, but note, using (A2.1) and (A3.1) that it can be shown that, in response to a fully anticipated change in the stock of money in period t:

$$\frac{\partial(N_t - N^*)}{\partial M_t} = \frac{\Psi_1\mu(1 - b_2)}{b_1 - b_2\mu^2}$$

and

$$\frac{\partial(N_{t-i} - N^*)}{\partial M_t} = \left(\frac{b_2\mu}{b_1}\right)^i \frac{\partial(N_t - N^*)}{\partial M_t}$$

$$\frac{\partial(N_{t+1} - N^*)}{\partial M_t} = \mu\frac{\partial(N_t - N^*)}{\partial M_t} < \frac{\partial(N_t - N^*)}{\partial M_t}$$

5. Next we move to the effects of an increase in the <u>growth rate</u> of money. Specifically, we assume

$$M_t - M_{t-1} = 0, \quad t = -\infty, \ldots, \tau$$

$$M_t - M_{t-1} = 1 \quad t = \tau+1, \ldots., \infty$$

Using (5.13) and (5.15) it is relatively straightforward to show

$$\frac{\partial(P_t - P_{t-1})}{\partial g} = \frac{\mu - b_2\mu^2}{b_1 - b_2\mu^2} < 1 \tag{A5.1}$$

$$\frac{\partial(P_{t-i} - P_{t-i-1})}{\partial g} = \left(\frac{b_2\mu}{b_1}\right)^i \frac{\partial(P_t - P_{t-1})}{\partial g} \qquad i = 1, 2, \ldots$$

$$\frac{\partial(P_{t+i} - P_{t+i-1})}{\partial g} = 1 - \frac{\mu^i(b_1 - \mu)}{b_1 - b_2\mu^2} \qquad i = 1, 2, \ldots.$$

where g is the change in the growth rate of money described above.

6. Looking at (A3.1), it is clear that inventories build up as the inflation rate increases; similarly from (A3.2), the real rate of interest falls continuously as the inflation rate increases. To study the behavior of output, use (A3.3); we leave it as an exercise to show, based on (A3.1) and (A5.1) that:

$$\text{(A6.1)} \qquad \frac{\partial N_t}{\partial g} = \frac{\Psi_1 \mu b_1 (1 - b_2)}{(b_1 - b_2\mu^2)(b_1 - b_2\mu)}$$

$$\frac{\partial N_{t-i}}{\partial g} = \left(\frac{b_2\mu}{b_1}\right)^i \frac{\partial N_t}{\partial g} \qquad i = 1, 2, ..$$

$$\frac{\partial N_{t+i}}{\partial g} = \frac{\partial N_{t+i-1}}{\partial g} + \frac{\mu^{i+1}\Psi_1(1 - b_2)}{b_1 - b_2\mu^2} \qquad i = 1, 2, ...,$$

Accordingly

$$\frac{\partial(N_{t+1} - N_t)}{\partial g} < \frac{\partial(N_t - N_{t-1})}{\partial g}$$